

A first course in linear algebra

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Consider the homogeneous system of linear equations $\langle \text{homosystem} | A \rangle$, and suppose that $\langle \text{vect} | u \rangle = \langle \text{colvector} | u_1$

u_2

u_3

\vdots

$u_n \rangle$ is one solution to the system of equations. Prove that $\langle \text{vect} | v \rangle = \langle \text{colvector} | 4u_1$

$4u_2$

$4u_3$

\vdots

$4u_n \rangle$ is also a solution to $\langle \text{homosystem} | A \rangle$.

Considere el Sistema Homogeno de Ecuaciones Lineales (Sistema Homogeneo | A). y supone que $\langle \text{vect} | u \rangle = \langle \text{colvector} | u_1$

u_2

u_3

\vdots

$u_n \rangle$ es una solucion del sistema de ecuaciones. Probar que $\langle \text{vect} | v \rangle = \langle \text{colvector} | 4u_1$

$4u_2$

$4u_3$

\vdots

$4u_n \rangle$ es tambien una solucion para (sistema Homogeneo | A)

SOLUCION:

Suppose that a single equation from this system (the i -th one) has the form,

Suponga esa, una ecuacion simple para este sistema (the i -th one) tiene la forma,

$$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \langle \text{dots} \rangle + a_{in}x_n = 0$$

Evaluate the left-hand side of this equation with the components of the proposed solution vector $\langle \text{vect} | v \rangle$,

Evaluando el lado izquierdo de esta ecuacion con los componenetes propuestos del vector solucion $\langle \text{vect} | v \rangle$,

$$\begin{aligned} a_{i1}(4u_1) + a_{i2}(4u_2) + a_{i3}(4u_3) + \dots + a_{in}(4u_n) \\ = 4a_{i1}u_1 + 4a_{i2}u_2 + 4a_{i3}u_3 + \dots + 4a_{in}u_n & \quad \text{Commutativity} \\ = 4(a_{i1}u_1 + a_{i2}u_2 + a_{i3}u_3 + \dots + a_{in}u_n) & \quad \text{Distributivity} \\ = 4(0) & \quad \langle \text{vect} | u \rangle \text{ solution to } \langle \text{homosystem} | A \rangle \\ = 0 & \quad \langle \text{vect} | u \rangle \text{ solucion para (Sistema Homogeneo | A)} \end{aligned}$$

So $\langle \text{vect} | v \rangle$ makes each equation true, and so is a solution to the system. Notice that this result is not true if we change $\langle \text{homosystem} | A \rangle$ from a homogeneous system to a non-homogeneous system. Can you create an example of a (non-homogeneous) system with a solution $\langle \text{vect} | u \rangle$ such that $\langle \text{vect} | v \rangle$ is not a solution?

entonces $(\text{vect} | \mathbf{v})$ hace de esta ecuacion verdadera, y entonces es una solucion para el sistema. Note que este resultado no es cierto si cambiamos (sistema Homogeneo|A) de un sistema Homogeneo a Un Sistema No Homogeneo. Puedes crear un ejemplo de un sistema(No homogeneo)con una solucion (vect | \mathbf{u}) tal que (vect | \mathbf{v}) no es una solucion?

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